<Contributed Talk 33>Spectral Theory of Nonlinear Fluid Flows Based on the Koopman operator

Mezic, Igor

IUTAM Symposium on 50 Years of Chaos : Applied and Theoretical
Kyoto, Japan, November 28 - December 2, 2011

Spectral Theory of Nonlinear Fluid Flows Based on the Koopman operator

Igor Mezić[1]

[1] *Department of Mechanical Engineering, University of Califonia, Santa Barbara, mezic@engineering.ucsb.edu*

Since we now understand that - barring a "blinding new technology" - the power of computers that will be available in the foreseeable future will not allow us to compute the details of physical interactions in many of the current problems in biological and physical sciences, such as molecular conformation or turbulence, the problem of *model reduction* has percolated to the top of the pile of open problems in Applied Mathematics. The number of different approaches in this direction is large, with some of the work relying on decompositions commonly used in probability theory - such as the Proper Orthogonal Decomposition (POD) (or Karhunen-Loeve, or Singular Value Decomposition) [6], and other projection methods such as the Mori-Zwanzig formalism and optimal prediction [3], the formalism that involves replacing higher-order nonlinear terms with stochastic processes [7], scale-separation and averaging methods, balanced truncation methods developed for linear control systems, operator-theoretic projection methods and coarse time-stepping methods. A good summary of a number of these is provided by Givon et al. [5]. In these approaches an analysis of how the dynamics on the attractor of the system that is being reduced affects the reduction is seldom found although attempts have been made [2]. An exception is the approach in [1] that uses directly the asymptotic dynamics on the attractor for projection and methods of Dellnitz and collaborators (see e.g. [4]) that utilizes properties of the Perron-Frobenius operator to reduce dynamics to a Markov chain.

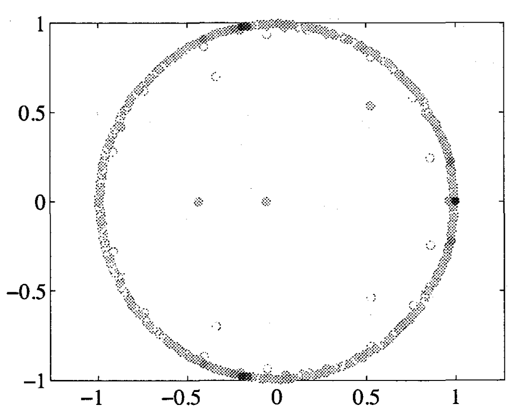
Here we discuss a model reduction theory based on spectral properties of the Koopman operator, develop an of-attractor decomposition into modes based on such spectral properties, and apply the theory to understanding of modal decomposition in complex fluid flows. The formalism in this paper (following our previous work in [9, 8]) is based on the adjoint of the Perron-Frobenius operator, the so-called Koopman operator. We review the application to complex fluid flow decomposition pursued in [10] applied to jet in the crossflow situation. The analysis involves using Wiener's generalized harmonic analysis, and a novel concept of generalized Laplace transform that is used to detect off-atractor, stable and unstable modes.

References

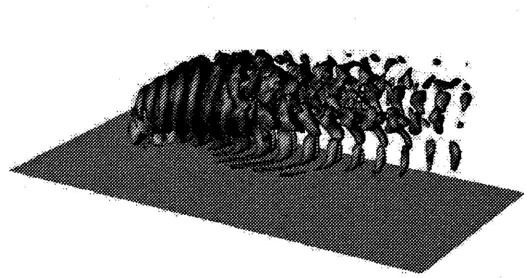
[1] A. Adrover and M. Giona. Modal reduction of pde models by means of snapshot archetypes. *Physica D*, 182:23–45, 2003.

[2] G. Berkooz. An observation on probability density equations, or, when do simulations reproduce statistics? *Nonlinearity*, 7:313–328, 1994.

[3] A. J. Chorin, O. H. Hald OH, and R. Kupferman. Optimal prediction and the mori-zwanzig representation of irreversible processes. *Proceedings of the National Academy of Sciences of the United States of America*, 97:2968–2973, 2000.



(a)



(b)

Figure 1: a) The spectrum of the Koopman operator for a jet-in-cross-flow obtained from a solution of Navier-Stokes equation. b) The mode corresponding to the lowest frequency mode of the Koopman operator. ([10])

[4] M. Dellnitz and O. Junge. On the approximation of complicated dynamical behavior. *SIAM Journal on Numerical Analysis*, 36(2):491–515, 1999.

[5] D. Givon, R. Kupferman, and A. Stuart. Extracting macroscopic dynamics: Model problems and algorithms. Warwick University Preprint, 2003.

[6] P. Holmes, J. L. Lumley, and G. Berkooz. *Turbulence, Coherent Structures, Dynamical Systems and Symmetry.* Cambridge University Press, Cambridge, 1996.

[7] A. J. Majda, I. Timofeyev, and E. V. Eijnden. Models for stochastic climate prediction. *Proceedings of the National Academy of Sciences*, 96:14687–14691, 1999.

[8] I. Mezić. Spectral Properties of Dynamical Systems, Model Reduction and Decompositions. *Nonlinear Dynamics*, 41(1):309–325, 2005.

[9] I. Mezić and A. Banaszuk. Comparison of complex systems. Accepted for publication in Physica D, 2004.

[10] C.W. Rowley, I. Mezić, S. Bagheri, P. Schlatter, and D.S. Henningson. Spectral analysis of nonlinear flows. *Journal of Fluid Mechanics*, 641:115–127, 2009.